

統計學·第 14 章 簡單線性迴歸

小主題練習卷·答案卷

Simple Linear Regression

時限：40 分鐘 | 總分：30 分 | 題數：3 題 | 每題標示配分與預估時間

姓名 _____ 學號 _____ 日期 _____

第 1 題

[10 分 · 約 · 配適 + 斜率檢定]

$x=1.5$; $y=3,5,4,8,10$; $\alpha = 0.05$ °

解答

$$\sum x = 15, \sum y = 30, \sum xy = 107, \sum x^2 = 55, \sum y^2 = 214, n = 5 \\ S_{xy} = 107 - \frac{15 \cdot 30}{5} = 17, S_{xx} = 55 - \frac{15^2}{5} = 10, S_{yy} = 214 - \frac{30^2}{5} = 34$$

$$(1) b_1 = \frac{17}{10} = 1.7, b_0 = \frac{30}{5} - 1.7 \cdot \frac{15}{5} = 0.9 \rightarrow \hat{y} = 0.9 + 1.7x$$

$$(2) SSE = 34 - \frac{17^2}{10} = 5.1, S_e = \sqrt{5.1/3} = 1.304, R^2 = \frac{17^2}{10 \cdot 34} = 0.85$$

$$(3) H_0 : \beta_1 = 0 ; H_1 : \beta_1 \neq 0 ; T = \frac{1.7}{1.304/\sqrt{10}} = \frac{1.7}{0.412} = 4.12, df = 3$$

$$(4) R = \{ |T| \geq t_{0.025,3} = 3.182 \} ; 4.12 \geq 3.182 \Rightarrow \text{Reject } H_0 : \text{廣告費與銷售額有顯著線性關係 } (R^2 = 0.85, \text{解釋 } 85\% \text{ 變異})$$

檢誤·易錯

迴歸線過 $(\bar{x}, \bar{y}) = (3, 6)$ 可驗 $b_0 \circ S_e \circ t$ 自由度 $n - 2 = 3 \circ R^2 = r^2 \circ$

第 2 題

[10 分 · 約 · 評估 $SSE/S_e/R^2$]

$n = 10$; $S_{xy} = 24, S_{xx} = 20, S_{yy} = 40$ °

解答

$$b_1 = \frac{24}{20} = 1.2 ; SSE = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = 40 - \frac{24^2}{20} = 40 - 28.8 = 11.2$$

$$S_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{11.2}{8}} = 1.183 ; R^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}} = \frac{576}{800} = 0.72$$

$R^2 = 0.72$: x 解釋了 y 約 72% 的變異 (解釋力佳) °

檢誤·易錯

S_e 分母 $n - 2 = 8$; $R^2 = S_{xy}^2 / (S_{xx}S_{yy})$, 等於 $1 - SSE/SST = 1 - 11.2/40 = 0.72$ 可互驗 °

第 3 題

[10 分 · 約 · 斜率 t 檢定]

$n = 12$; $b_1 = 0.30, S_e = 0.90, S_{xx} = 25$; $\alpha = 0.05$ °

解答

(1) $H_0 : \beta_1 = 0$ (2) $H_1 : \beta_1 \neq 0$ (3) $T = \frac{b_1}{S_e/\sqrt{S_{xx}}}$, $df = 10$ (4) $\alpha = 0.05$

(5) $R = \{|T| \geq t_{0.025,10} = 2.228\}$

(6) $T = \frac{0.30}{0.90/\sqrt{25}} = \frac{0.30}{0.18} = 1.67$

(7) $1.67 < 2.228 \Rightarrow$ **Do not reject** H_0 (8) 無足夠證據說 x 與 y 有線性關係 (不宜用此線預測)。

檢誤·易錯

$S_e/\sqrt{S_{xx}} = 0.90/5 = 0.18$ 。本題不顯著 \rightarrow **不拒絕** (別自動寫拒絕)；不顯著時用迴歸線預測意義不大。